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# QCD corrections to the decay $H^+ \rightarrow t\bar{b}$ in the Minimal Supersymmetric Standard Model

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## Abstract

We present a complete calculation of the  $\mathcal{O}(\alpha_s)$  QCD corrections to the width of the decay  $H^+ \rightarrow t\bar{b}$  within the Minimal Supersymmetric Standard Model. We find that the QCD corrections are quite important, and that the supersymmetric QCD corrections (due to gluino,  $\tilde{t}$  and  $\tilde{b}$  exchange) can be comparable to or even larger than the standard QCD corrections in a large region of the supersymmetric parameter space. This is mainly due to the effect of large left-right mixings of stop ( $\tilde{t}$ ) and sbottom ( $\tilde{b}$ ). This could significantly affect the phenomenology of the  $H^+$  search.

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# 1 Introduction

The existence of a charged Higgs boson  $H^+$  would be a clear indication that the Standard Model must be extended. For example, the Minimal Supersymmetric Standard Model (MSSM) [1] with two Higgs doublets predicts the existence of five physical Higgs bosons  $h^0, H^0, A^0$ , and  $H^\pm$  [2, 3]. If all supersymmetric (SUSY) particles are heavy enough,  $H^+$  decays dominantly into  $t\bar{b}$  above the  $t\bar{b}$  threshold [2, 4]. In refs. [5, 6] all decay modes of  $H^+$  including the SUSY-particle modes were studied in the case that the SUSY-particles are relatively light: it was found that the  $t\bar{b}$  mode remains important even in this case (though the  $\tilde{t}\tilde{b}$  mode can be dominant in a wide range of the MSSM parameters). Thus it is important to calculate the QCD corrections to the  $t\bar{b}$  mode as they could significantly affect the phenomenology of the  $H^+$  search. The standard QCD corrections to the  $t\bar{b}$  mode were already calculated [7]: they can be large (+10% to -50%). There also exist calculations of the SUSY-QCD corrections within the MSSM [8, 9]. However, in ref. [8] the squark-mixing was neglected. The calculation in ref. [9] is incomplete as the wave function and mass renormalizations were omitted. A calculation of SUSY-QCD corrections to the related process  $t \rightarrow H^+ b$  was done recently in ref. [10].

In this paper we present a complete calculation of the  $\mathcal{O}(\alpha_s)$  QCD corrections to the width of  $H^+ \rightarrow t\bar{b}$  within the MSSM. We include the left-right mixings of both the  $\tilde{t}_{L,R}$  squarks and the  $\tilde{b}_{L,R}$  squarks. We adopt the on-shell renormalization scheme.

## 2 QCD one-loop contributions

The one-loop corrected amplitude of the decay  $H^+(p) \rightarrow t(k_t)\bar{b}(k_{\bar{b}})$  ( $p = k_t + k_{\bar{b}}$ ) can be written as

$$\mathcal{M} = i\bar{t}(Y_1 P_R + Y_2 P_L)b \quad (1)$$

with  $P_{R,L} = \frac{1}{2}(1 \pm \gamma_5)$  and the one-loop corrected couplings:

$$Y_i = y_i + \delta Y_i^{(g)} + \delta Y_i^{(\tilde{g})} \quad (i = 1, 2), \quad (2)$$

where  $y_i$  are the tree-level couplings corresponding to Fig. 1a:

$$\begin{aligned} y_1 &= \frac{g}{\sqrt{2}m_W} m_b \tan \beta = h_b \sin \beta, \\ y_2 &= \frac{g}{\sqrt{2}m_W} m_t \cot \beta = h_t \cos \beta, \end{aligned} \quad (3)$$

with  $g$  being the SU(2) coupling.  $\delta Y_i^{(g)}$  and  $\delta Y_i^{(\tilde{g})}$  are the contributions from gluon and gluino exchanges, respectively (as shown in Figs. 1b and 1c).

The tree-level decay width is given by:

$$\Gamma^{\text{tree}}(H^+ \rightarrow t\bar{b}) = \frac{N_C \kappa}{16\pi m_{H^+}^3} [(m_{H^+}^2 - m_t^2 - m_b^2)(y_1^2 + y_2^2) - 4m_t m_b y_1 y_2], \quad (4)$$

where  $\kappa = \kappa(m_{H^+}^2, m_t^2, m_b^2)$ ,  $\kappa(x, y, z) \equiv ((x - y - z)^2 - 4yz)^{1/2}$ , and  $N_C = 3$ .

The vertex corrections due to gluon and gluino exchanges at the vertex (Fig. 1b),  $\delta Y_i^{(v,g)}$  and  $\delta Y_i^{(v,\tilde{g})}$ , respectively, are given by:

$$\begin{aligned} \delta(Y_1 P_R + Y_2 P_L)^{(v,g)} &= \frac{\alpha_s C_F}{4\pi} \left\{ 2[B_0(m_t^2, 0, m_t^2) + B_0(m_b^2, 0, m_b^2) - r \right. \\ &\quad \left. -(m_{H^+}^2 - m_t^2 - m_b^2)C_0(\lambda^2, m_t^2, m_b^2)](y_1 P_R + y_2 P_L) \right. \\ &\quad \left. - 2m_t C_1(\lambda^2, m_t^2, m_b^2)[(m_t y_1 + m_b y_2)P_R + (m_t y_2 + m_b y_1)P_L] \right. \\ &\quad \left. - 2m_b C_2(\lambda^2, m_t^2, m_b^2)[(m_t y_2 + m_b y_1)P_R + (m_t y_1 + m_b y_2)P_L] \right\}, \\ \delta(Y_1 P_R + Y_2 P_L)^{(v,\tilde{g})} &= \frac{\alpha_s C_F}{4\pi} \left\{ 2G_{ij} \left[ -m_{\tilde{g}} C_0(m_{\tilde{g}}^2, m_{\tilde{t}_i}^2, m_{\tilde{b}_j}^2) \{(\alpha_{LR})_{ij} P_R + (\alpha_{RL})_{ij} P_L \right. \right. \\ &\quad \left. + m_t C_1(m_{\tilde{g}}^2, m_{\tilde{t}_i}^2, m_{\tilde{b}_j}^2) \{(\alpha_{LL})_{ij} P_L + (\alpha_{RR})_{ij} P_R \} \right. \\ &\quad \left. + m_b C_2(m_{\tilde{g}}^2, m_{\tilde{t}_i}^2, m_{\tilde{b}_j}^2) \{(\alpha_{LL})_{ij} P_R + (\alpha_{RR})_{ij} P_L \} \right] \right\}. \end{aligned} \quad (5)$$

with  $C_F = 4/3$  and

$$\begin{aligned} \alpha_{LL} &= \begin{pmatrix} \cos \theta_{\tilde{t}} \cos \theta_{\tilde{b}} & -\cos \theta_{\tilde{t}} \sin \theta_{\tilde{b}} \\ -\sin \theta_{\tilde{t}} \cos \theta_{\tilde{b}} & \sin \theta_{\tilde{t}} \sin \theta_{\tilde{b}} \end{pmatrix}, \quad \alpha_{LR} = \begin{pmatrix} -\cos \theta_{\tilde{t}} \sin \theta_{\tilde{b}} & -\cos \theta_{\tilde{t}} \cos \theta_{\tilde{b}} \\ \sin \theta_{\tilde{t}} \sin \theta_{\tilde{b}} & \sin \theta_{\tilde{t}} \cos \theta_{\tilde{b}} \end{pmatrix}, \\ \alpha_{RL} &= \begin{pmatrix} -\sin \theta_{\tilde{t}} \cos \theta_{\tilde{b}} & \sin \theta_{\tilde{t}} \sin \theta_{\tilde{b}} \\ -\cos \theta_{\tilde{t}} \cos \theta_{\tilde{b}} & \cos \theta_{\tilde{t}} \sin \theta_{\tilde{b}} \end{pmatrix}, \quad \alpha_{RR} = \begin{pmatrix} \sin \theta_{\tilde{t}} \sin \theta_{\tilde{b}} & \sin \theta_{\tilde{t}} \cos \theta_{\tilde{b}} \\ \cos \theta_{\tilde{t}} \sin \theta_{\tilde{b}} & \cos \theta_{\tilde{t}} \cos \theta_{\tilde{b}} \end{pmatrix}. \end{aligned} \quad (6)$$

$G_{ij}$  are the tree-level couplings of  $H^+$  to  $\tilde{t}_i \bar{b}_j$  ( $i, j = 1, 2$ ) reading:

$$G_{ij} = \frac{g}{\sqrt{2}m_W} R^{\tilde{t}} \begin{pmatrix} m_b^2 \tan \beta + m_t^2 \cot \beta - m_W^2 \sin 2\beta & m_b(A_b \tan \beta + \mu) \\ m_t(A_t \cot \beta + \mu) & 2m_t m_b / \sin 2\beta \end{pmatrix} (R^{\tilde{b}})^\dagger. \quad (7)$$

Here  $R^{\tilde{q}}$  ( $\tilde{q} = \tilde{t}$  or  $\tilde{b}$ ) is the  $\tilde{q}$ -mixing matrix

$$R_{i\alpha}^{\tilde{q}} = \begin{pmatrix} \cos \theta_{\tilde{q}} & \sin \theta_{\tilde{q}} \\ -\sin \theta_{\tilde{q}} & \cos \theta_{\tilde{q}} \end{pmatrix} \quad (i = 1, 2; \alpha = L, R) \quad (8)$$

relating the squark states  $\tilde{q}_L$  and  $\tilde{q}_R$  to the mass-eigenstates  $\tilde{q}_1$  and  $\tilde{q}_2$  ( $m_{\tilde{q}_1} < m_{\tilde{q}_2}$ ):  $\tilde{q}_i = R_{i\alpha}^{\tilde{q}} \tilde{q}_\alpha$ .  $R^{\tilde{q}}$  diagonalizes the squark mass matrix [3]:

$$\begin{pmatrix} m_{LL}^2 & m_{LR}^2 \\ m_{RL}^2 & m_{RR}^2 \end{pmatrix} = (R^{\tilde{q}})^\dagger \begin{pmatrix} m_{\tilde{q}_1}^2 & 0 \\ 0 & m_{\tilde{q}_2}^2 \end{pmatrix} R^{\tilde{q}}, \quad (9)$$

where

$$m_{LL}^2 = M_Q^2 + m_q^2 + m_Z^2 \cos 2\beta (I_q - Q_q \sin^2 \theta_W), \quad (10)$$

$$m_{RR}^2 = M_{\{\tilde{U}, \tilde{D}\}}^2 + m_q^2 + m_Z^2 \cos 2\beta Q_q \sin^2 \theta_W, \quad (11)$$

$$m_{LR}^2 = m_{RL}^2 = \begin{cases} m_t(A_t - \mu \cot \beta) & (\tilde{q} = \tilde{t}) \\ m_b(A_b - \mu \tan \beta) & (\tilde{q} = \tilde{b}) \end{cases}. \quad (12)$$

As usually, we introduce a gluon mass  $\lambda$  for the regularization of the infrared divergence. Here we define the functions  $B_0$ ,  $B_1$ ,  $C_0$ ,  $C_1$ , and  $C_2$  as in [11, 12]:

$$\begin{aligned} [B_0, k^\mu B_1] (k^2, m_0^2, m_1^2) &= \int \frac{d^D q}{i\pi^2} \frac{[1, q^\mu]}{(q^2 - m_0^2)((q+k)^2 - m_1^2)} \\ [C_0, k_t^\mu C_1 - k_{\bar{b}}^\mu C_2] (m_0^2, m_1^2, m_2^2) &= \int \frac{d^D q}{i\pi^2} \frac{[1, q^\mu]}{(q^2 - m_0^2)((q+k_t)^2 - m_1^2)((q-k_{\bar{b}})^2 - m_2^2)}. \end{aligned} \quad (13)$$

Here  $k_t$  and  $k_{\bar{b}}$  are the external momenta of  $t$  and  $\bar{b}$ , respectively. The parameter  $r$  in eq. (5) and following equations shows the dependence on the regularization:  $r = 1$  for dimensional regularization and  $r = 0$  for the dimensional reduction (DR) [13]. The dependence on  $r$ , however, disappears in our final result.

Now we turn to the quark wave-function renormalization due to the graphs of Fig. 1c. The two-point vertex function for  $\bar{q}q$  can be written as:

$$\not{k}(1 + \Pi_L^q(k^2)P_L + \Pi_R^q(k^2)P_R) - (m_q + \Sigma_L^q(k^2)P_L + \Sigma_R^q(k^2)P_R). \quad (14)$$

Here we have  $\Sigma_L^q(k^2) = \Sigma_R^q(k^2) \equiv \Sigma^q(k^2)$ . The correction to the amplitude from the wave-function renormalization has the form:

$$\begin{aligned} \delta(Y_1 P_R + Y_2 P_L)^{(w)} &= -\frac{1}{2}(\Pi_L^t(m_t^2) + \Pi_R^b(m_b^2))y_1 P_R - \frac{1}{2}(\Pi_R^t(m_t^2) + \Pi_L^b(m_b^2))y_2 P_L \\ &+ (m_t \dot{\Sigma}^t(m_t^2) - m_t^2 \dot{\Pi}^t(m_t^2) + m_b \dot{\Sigma}^b(m_b^2) - m_b^2 \dot{\Pi}^b(m_b^2))(y_1 P_R + y_2 P_L), \end{aligned} \quad (15)$$

with  $\dot{\Pi}^q \equiv \frac{1}{2}(\dot{\Pi}_L^q + \dot{\Pi}_R^q)$  and  $\dot{X} \equiv \frac{dX}{dk^2}$ . The explicit calculation yields:

$$\begin{aligned} (\Pi_L^q(k^2)P_L + \Pi_R^q(k^2)P_R)^{(g)} &= \frac{\alpha_s C_F}{4\pi} [-2B_1(k^2, m_q^2, \lambda^2) - r], \\ (\Pi_L^q(k^2)P_L + \Pi_R^q(k^2)P_R)^{(\tilde{g})} &= -\frac{\alpha_s C_F}{4\pi} [2(\cos^2 \theta_{\tilde{q}} P_L + \sin^2 \theta_{\tilde{q}} P_R) B_1(k^2, m_{\tilde{g}}^2, m_{\tilde{q}_1}^2) \\ &\quad + 2(\sin^2 \theta_{\tilde{q}} P_L + \cos^2 \theta_{\tilde{q}} P_R) B_1(k^2, m_{\tilde{g}}^2, m_{\tilde{q}_2}^2)], \end{aligned} \quad (16)$$

$$\Sigma^q(k^2)^{(g)} = \frac{\alpha_s C_F}{4\pi} m_q [4B_0(k^2, m_q^2, \lambda^2) - 2r], \quad (17)$$

$$\Sigma^q(k^2)^{(\tilde{g})} = \frac{\alpha_s C_F}{4\pi} [m_{\tilde{g}} \sin 2\theta_{\tilde{q}} (B_0(k^2, m_{\tilde{g}}^2, m_{\tilde{q}_1}^2) - B_0(k^2, m_{\tilde{g}}^2, m_{\tilde{q}_2}^2))]. \quad (18)$$

Finally, there are additional corrections  $\delta Y_i^{(0)}$  by the renormalization of the quark masses in the couplings of eq. (3) (In the  $\overline{\text{DR}}$  scheme equivalent corrections are necessary as one takes the physical masses of the quarks as input):

$$\begin{aligned} \delta Y_1^{(0)} = \delta y_1 &= \frac{g}{\sqrt{2}m_W} \delta m_b \tan \beta, \\ \delta Y_2^{(0)} = \delta y_2 &= \frac{g}{\sqrt{2}m_W} \delta m_t \cot \beta, \end{aligned} \quad (19)$$

$$\begin{aligned} \text{with } \delta m_q &= \delta m_q^{(g)} + \delta m_q^{(\tilde{g})}, \\ \delta m_q^{(g)} &= -\frac{\alpha_s C_F}{4\pi} [2m_q (B_0(m_q^2, 0, m_q^2) - B_1(m_q^2, 0, m_q^2) - \frac{r}{2})], \text{ and} \\ \delta m_q^{(\tilde{g})} &= -\frac{\alpha_s C_F}{4\pi} [\sin 2\theta_{\tilde{q}} m_{\tilde{g}} (B_0(m_q^2, m_{\tilde{g}}^2, m_{\tilde{q}_1}^2) - B_0(m_q^2, m_{\tilde{g}}^2, m_{\tilde{q}_2}^2)) \\ &\quad + m_q (B_1(m_q^2, m_{\tilde{g}}^2, m_{\tilde{q}_1}^2) + B_1(m_q^2, m_{\tilde{g}}^2, m_{\tilde{q}_2}^2))]. \end{aligned} \quad (20)$$

Taking all contributions eqs. (3, 5, 15, 19) together, we get the one-loop corrected couplings  $Y_i = y_i + \delta Y_i = y_i + \delta Y_i^{(0)} + \delta Y_i^{(v)} + \delta Y_i^{(w)}$  with contributions to  $\delta Y_i^{(0),(v),(w)}$  from gluon and gluino exchanges (see eq. (2)). It can be readily seen that they are ultraviolet finite but still infrared divergent. The one-loop corrected decay width to  $\mathcal{O}(\alpha_s)$  is then given by

$$\begin{aligned} \Gamma(H^+ \rightarrow t\bar{b}) &= \frac{N_C \kappa}{16\pi m_{H^+}^3} \left[ (m_{H^+}^2 - m_t^2 - m_b^2) (y_1^2 + y_2^2 + 2y_1 \text{Re}(\delta Y_1) + 2y_2 \text{Re}(\delta Y_2)) \right. \\ &\quad \left. - 4m_t m_b (y_1 y_2 + y_1 \text{Re}(\delta Y_2) + y_2 \text{Re}(\delta Y_1)) \right]. \end{aligned} \quad (21)$$

### 3 Inclusion of the gluon emission

For the cancellation of the infrared divergencies ( $\lambda \rightarrow 0$ ) it is necessary to include the  $\mathcal{O}(\alpha_s)$  contribution from real gluon emission as shown in Fig. 1d.

The decay width of  $H^+ \rightarrow t + \bar{b} + g$  is given by

$$\begin{aligned}\Gamma(H^+ \rightarrow t\bar{b}g) &= \frac{\alpha_s C_F N_C}{4\pi^2 m_{H^+}} \left[ (y_1^2 + y_2^2) \{ J_1 - (m_{H^+}^2 - m_t^2 - m_b^2) J_2 \right. \\ &\quad \left. + (m_{H^+}^2 - m_t^2 - m_b^2)^2 I_{12} \} + 4m_t m_b y_1 y_2 \{ J_2 - (m_{H^+}^2 - m_t^2 - m_b^2) I_{12} \} \right],\end{aligned}\quad (22)$$

with the integrals

$$\begin{aligned}I_{12} &= \frac{1}{4m_{H^+}^2} \left[ -2 \ln \left( \frac{\lambda m_{H^+} m_t m_b}{\kappa^2} \right) \ln \beta_0 + 2 \ln^2 \beta_0 - \ln^2 \beta_1 - \ln^2 \beta_2 \right. \\ &\quad \left. + 2 \text{Sp}(1 - \beta_0^2) - \text{Sp}(1 - \beta_1^2) - \text{Sp}(1 - \beta_2^2) \right] \quad (23)\end{aligned}$$

$$\begin{aligned}J_1 &= \frac{1}{2} I_1^2 + \frac{1}{2} I_2^1 + I = -\frac{1}{2} I_1^0 - \frac{1}{2} I_2^0 \\ &= \frac{1}{8m_{H^+}^2} \left[ (\kappa^2 + 6m_t^2 m_b^2) \ln \beta_0 - \frac{3}{2} \kappa (m_{H^+}^2 - m_t^2 - m_b^2) \right] \quad (24)\end{aligned}$$

$$\begin{aligned}J_2 &= m_t^2 I_{11} + m_b^2 I_{22} + I_1 + I_2 \\ &= -\frac{1}{4m_{H^+}^2} \left[ 2\kappa \ln \left( \frac{\lambda m_{H^+} m_t m_b}{\kappa^2} \right) + 4\kappa + (m_{H^+}^2 + m_t^2 + m_b^2) \ln \beta_0 \right. \\ &\quad \left. + 2m_t^2 \ln \beta_1 + 2m_b^2 \ln \beta_2 \right]. \quad (25)\end{aligned}$$

Here

$$\begin{aligned}\beta_0 &\equiv \frac{m_{H^+}^2 - m_t^2 - m_b^2 + \kappa}{2m_t m_b}, \quad \beta_1 \equiv \frac{m_{H^+}^2 - m_t^2 + m_b^2 - \kappa}{2m_{H^+} m_b}, \\ \beta_2 &\equiv \frac{m_{H^+}^2 + m_t^2 - m_b^2 - \kappa}{2m_{H^+} m_t}, \quad \text{Sp}(x) = - \int_0^x \frac{dt}{t} \ln(1-t),\end{aligned}\quad (26)$$

and  $\kappa = \kappa(m_{H^+}^2, m_t^2, m_b^2)$ . The definitions and the explicit forms of the  $I$ 's are given in [11].

The one-loop corrected decay width to  $\mathcal{O}(\alpha_s)$  including the real gluon emission can be written as:

$$\begin{aligned}\Gamma^{\text{corr}}(H^+ \rightarrow t\bar{b} + t\bar{b}g) &\equiv \Gamma(H^+ \rightarrow t\bar{b}) + \Gamma(H^+ \rightarrow t\bar{b}g) \\ &= \Gamma^{\text{tree}}(H^+ \rightarrow t\bar{b}) + \delta\Gamma(\text{gluon}) + \delta\Gamma(\text{gluino}),\end{aligned}\quad (27)$$

with  $\Gamma^{\text{tree}}$  given by eq. (4), and

$$\begin{aligned}\delta\Gamma(\text{gluon}) &= \frac{N_C\kappa}{16\pi m_{H^+}^3} \left[ 2(m_{H^+}^2 - m_t^2 - m_b^2) \left( y_1 \text{Re}(\delta Y_1^{(g)}) + y_2 \text{Re}(\delta Y_2^{(g)}) \right) \right. \\ &\quad \left. - 4m_t m_b \left( y_1 \text{Re}(\delta Y_2^{(g)}) + y_2 \text{Re}(\delta Y_1^{(g)}) \right) \right] + \Gamma(H^+ \rightarrow t\bar{b}g),\end{aligned}\quad (28)$$

$$\begin{aligned}\delta\Gamma(\text{gluino}) &= \frac{N_C\kappa}{16\pi m_{H^+}^3} \left[ 2(m_{H^+}^2 - m_t^2 - m_b^2) \left( y_1 \text{Re}(\delta Y_1^{(\tilde{g})}) + y_2 \text{Re}(\delta Y_2^{(\tilde{g})}) \right) \right. \\ &\quad \left. - 4m_t m_b \left( y_1 \text{Re}(\delta Y_2^{(\tilde{g})}) + y_2 \text{Re}(\delta Y_1^{(\tilde{g})}) \right) \right].\end{aligned}\quad (29)$$

We have checked that the corrected width of eq. (27) is infrared finite.

## 4 Numerical Results and Discussion

We now turn to the numerical evaluation of the corrected width eq.(27). As the standard QCD corrections have already been calculated [7], it is interesting here to study the influence of the gluino (and  $\tilde{t}_i, \tilde{b}_j$ ) exchange corrections  $\delta\Gamma(\text{gluino})$ . The whole analysis depends on the following parameters defined at the weak scale:  $m_{H^+}, \tan\beta, \mu, A_t, A_b, M_{\tilde{Q}}, M_{\tilde{U}}, M_{\tilde{D}}$ , and  $m_{\tilde{g}}$ . For simplicity we assume  $M_{\tilde{Q}} = M_{\tilde{U}} = M_{\tilde{D}}$  and  $A_t = A_b \equiv A$ . We have found that our final results are rather insensitive to these assumptions. We take  $m_t = 180$  GeV,  $m_b = 5$  GeV,  $m_W = 80$  GeV,  $m_Z = 91.2$  GeV,  $\sin^2\theta_W = 0.23$ ,  $g^2/(4\pi) = \alpha_2 = \alpha/\sin^2\theta_W = 0.0337$  and  $\alpha_s = \alpha_s(m_{H^+})$ . We use  $\alpha_s(Q) = 12\pi/\{(33 - 2n_f)\ln(Q^2/\Lambda_{n_f}^2)\}$  with  $\alpha_s(m_Z) = 0.12$  and the number of quark flavors  $n_f = 5(6)$  for  $m_b < Q \leq m_t$  (for  $Q > m_t$ ).

In Fig. 2 we show the dependence of  $\delta\Gamma(\text{gluino})$  as a function of  $A$  and  $M_{\tilde{Q}}$  for  $\tan\beta = 2$  (a) and 12 (b), and  $(m_{H^+}, m_{\tilde{g}}, \mu) = (400, 550, 300)$  (GeV). We see that the size of the SUSY-QCD correction  $\delta\Gamma(\text{gluino})$  can be large going up to  $\sim 50\%$  and that it can be comparable to or even larger than the standard QCD correction  $\delta\Gamma(\text{gluon})$  in a large parameter region. For fixed  $\tan\beta$ ,  $\delta\Gamma(\text{gluino})$  has a strong dependence on the parameters  $M_{\tilde{Q}}$  and  $A$  which determine the masses and couplings of  $\tilde{t}_{1,2}$  and  $\tilde{b}_{1,2}$ .  $\delta\Gamma(\text{gluino})$  is smaller for larger masses of  $\tilde{t}_1$  and  $\tilde{b}_1$ : for  $\tan\beta = 2$ , the correction due to  $\delta\Gamma(\text{gluino})$  is about  $-15\%$  for  $(M_{\tilde{Q}}, A) = (100 \text{ GeV}, 300 \text{ GeV})$

(where  $m_{\tilde{t}_1} \simeq 119$  GeV, and  $m_{\tilde{b}_1} \simeq 98$  GeV), but it is still  $\sim -5\%$  for larger squark masses  $(M_{\tilde{Q}}, A) = (400$  GeV, 300 GeV) (where  $m_{\tilde{t}_1} \simeq 405$  GeV, and  $m_{\tilde{b}_1} \simeq 399$  GeV). This tendency is consistent with the decoupling theorem for the MSSM. Notice also the different behaviour for  $\tan \beta = 2$  and  $\tan \beta = 12$ .

In Fig. 3 we show the  $m_{H^+}$  dependence of  $\Gamma^{\text{tree}}$ ,  $\Gamma^{\text{tree}} + \delta\Gamma(\text{gluon})$ , and  $\Gamma^{\text{corr}} = \Gamma^{\text{tree}} + \delta\Gamma(\text{gluon}) + \delta\Gamma(\text{gluino})$  for  $\tan \beta = 2$  (a) and 12 (b), and  $(m_{\tilde{g}}, \mu, M_{\tilde{Q}}, A) = (400, -300, 200, 200)$  (GeV). The parameter values correspond to fixed stop and sbottom masses:  $m_{\tilde{t}_1} = 90$  GeV,  $m_{\tilde{t}_2} = 366$  GeV,  $m_{\tilde{b}_1} = 193$  GeV, and  $m_{\tilde{b}_2} = 213$  GeV (for  $\tan \beta = 2$ ) and  $m_{\tilde{t}_1} = 173$  GeV,  $m_{\tilde{t}_2} = 333$  GeV,  $m_{\tilde{b}_1} = 152$  GeV, and  $m_{\tilde{b}_2} = 247$  GeV (for  $\tan \beta = 12$ ). (Note that for  $m_{\tilde{g}} = 400$  GeV the D0 mass limit of the mass-degenerate squarks of five flavors (excluding  $\tilde{t}_{1,2}$ ) is  $m_{\tilde{q}} \gtrsim 140$  GeV [14].) We see again that the correction  $\delta\Gamma(\text{gluino})$  can be quite large and that it is comparable to or even larger than  $\delta\Gamma(\text{gluon})$  in a large region. Quite generally, the corrections  $\delta\Gamma(\text{gluon})$  and  $\delta\Gamma(\text{gluino})$  are bigger for larger  $\tan \beta$ , but it can happen that they partly cancel each other. The correction  $\delta\Gamma(\text{gluon})$  has already been calculated in [7]. Our results on  $\delta\Gamma(\text{gluon})$  agree numerically with ref. 7 within 10%.

In Fig. 4 we show a contour-plot for  $\frac{\delta\Gamma(\text{gluino})}{\Gamma^{\text{corr}}}$  in the  $\tan \beta - m_{\tilde{g}}$  plane for  $(m_{H^+}, \mu, M_{\tilde{Q}}, A) = (400, -300, 250, 300)$  GeV. This correction rises with increasing  $\tan \beta$ , going up to 50%! Concerning the  $m_{\tilde{g}}$  dependence,  $\frac{\delta\Gamma(\text{gluino})}{\Gamma^{\text{corr}}}$  increases up to  $m_{\tilde{g}} = 300 - 450$  GeV and then decreases gradually as  $m_{\tilde{g}}$  increases. It is striking that even for a large gluino mass ( $\sim 1$  TeV)  $\frac{\delta\Gamma(\text{gluino})}{\Gamma^{\text{corr}}}$  is larger than 10% for  $\tan \beta \gtrsim 3$ . From Figs. 2 and 4 we see that the correction  $\delta\Gamma(\text{gluino})$  decreases much faster for increasing  $M_{\tilde{Q}}$  than for increasing  $m_{\tilde{g}}$ .

In Fig. 5 we show contour lines of  $\delta\Gamma(\text{gluino})$  in the  $\mu - A$  plane for  $\tan \beta = 2$  (a) and 12 (b), and  $(m_{H^+}, m_{\tilde{g}}, M_{\tilde{Q}}) = (400, 550, 300)$  GeV. This correction has a strong dependence on  $\mu$  and a significant dependence on  $A$ . We have found that the

$A$  dependence for  $\tan\beta = 1$  is much stronger than that for  $\tan\beta = 2$ .

The reason for the large contribution of  $\delta\Gamma(\text{gluino})$  as compared to  $\delta\Gamma(\text{gluon})$  is the following: The vertex-correction part of the gluino-exchange [gluon-exchange] corrections (see Fig. 1b and eq. (5)) is proportional to the  $H^+\tilde{t}\tilde{b}$  coupling [ $H^+\bar{t}b$  coupling] which is essentially  $\sim (A_t + \mu \tan\beta)h_t \cos\beta + (A_b + \mu \cot\beta)h_b \sin\beta$  [ $\sim h_t \cos\beta + h_b \sin\beta$ ]. Hence the vertex-correction part of the gluino-exchange corrections  $\delta\Gamma(\text{gluino})$  can be strongly enhanced relative to that of the gluon-exchange corrections  $\delta\Gamma(\text{gluon})$  in the case the  $\tilde{q}$ -mixing parameters  $A$  and  $\mu$  are large. In this case  $\tilde{t}_1$  and  $\tilde{b}_1$  tend to be light due to a large mass-splitting. Note that the  $\tilde{b}$ -mixing effect plays a very important role for large  $\tan\beta$ .

## 5 Conclusion

Summarizing, we have performed a complete calculation of the  $\mathcal{O}(\alpha_s)$  QCD corrections to the width of  $H^+ \rightarrow t\bar{b}$  within the MSSM. We have found that the QCD corrections are quite important. A detailed numerical analysis has shown that the SUSY-QCD corrections (due to gluino,  $\tilde{t}$  and  $\tilde{b}$  exchanges) can be comparable to or even larger than the standard QCD corrections in a large region of the MSSM parameter space; here the mixings of  $\tilde{t}_L - \tilde{t}_R$  and  $\tilde{b}_L - \tilde{b}_R$  play a crucial role. This could significantly affect the phenomenology of the  $H^+$  search.

After having finished this study, we have been informed on a recent paper [15] dealing with the same subject.

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## Figure Captions

Fig. 1 All diagrams relevant for the calculation of the  $\mathcal{O}(\alpha_s)$  QCD corrections to the width of  $H^+ \rightarrow t\bar{b}$  in the MSSM.

Fig. 2 Contour lines of  $\delta\Gamma(\text{gluino})$  (GeV) in the  $A - M_{\tilde{Q}}$  plane for  $\tan\beta = 2$  (a) and 12 (b), and  $(m_{H^+}, m_{\tilde{g}}, \mu) = (400, 550, 300)$  (GeV). For these parameter values one has  $(\Gamma^{\text{tree}} \text{ (GeV)}, \delta\Gamma(\text{gluon}) \text{ (GeV)}) = (4.10, 0.31)$  and  $(1.91, -0.66)$  for Figs. 2a and 2b, respectively. The shaded area is excluded by the LEP bounds  $m_{\tilde{t}_1, \tilde{b}_1} \gtrsim 45$  GeV. Note that for  $m_{\tilde{g}} \simeq 550$  GeV one has no squark mass bound from D0 experiment [14].

Fig. 3  $m_{H^+}$  dependence of  $\Gamma^{\text{tree}}$  (dashed line),  $\Gamma^{\text{tree}} + \delta\Gamma(\text{gluon})$  (dot-dashed line), and  $\Gamma^{\text{corr}} = \Gamma^{\text{tree}} + \delta\Gamma(\text{gluon}) + \delta\Gamma(\text{gluino})$  (solid line) for  $\tan\beta = 2$  (a) and 12 (b), and  $(m_{\tilde{g}}, \mu, M_{\tilde{Q}}, A) = (400, -300, 200, 200)$  (GeV).

Fig. 4 Contour lines of  $\delta\Gamma(\text{gluino})/\Gamma^{\text{corr}}$  in the  $\tan\beta - m_{\tilde{g}}$  plane for  $(m_{H^+}, \mu, M_{\tilde{Q}}, A) = (400, -300, 250, 300)$  (GeV). The area below the dotted line is excluded by the LEP limit  $m_{\tilde{\chi}_1^+} \gtrsim 45$  GeV (assuming  $m_{\tilde{g}} = (\alpha_s/\alpha_2)M_2 \simeq 3.56M_2$ ), where  $\alpha_2 = g^2/(4\pi)$ ,  $M_2$  is the SU(2) gaugino mass, and  $m_{\tilde{\chi}_1^+}$  is the lighter chargino mass.

Fig. 5 Contour lines of  $\delta\Gamma(\text{gluino})$  (GeV) in the  $\mu - A$  plane for  $\tan\beta = 2$  (a) and 12 (b), and  $(m_{H^+}, m_{\tilde{g}}, M_{\tilde{Q}}) = (400, 550, 300)$  GeV. For these parameter values one has  $(\Gamma^{\text{tree}} \text{ (GeV)}, \delta\Gamma(\text{gluon}) \text{ (GeV)}) = (4.10, 0.31)$  and  $(1.91, -0.66)$  for Figs. 5a and 5b, respectively. The shaded area is excluded by the LEP limits  $m_{\tilde{t}_1, \tilde{b}_1, \tilde{\chi}_1^+} \gtrsim 45$  GeV. For  $m_{\tilde{g}} \simeq 550$  GeV one has no  $m_{\tilde{q}}$  limit from the D0 experiment [14].

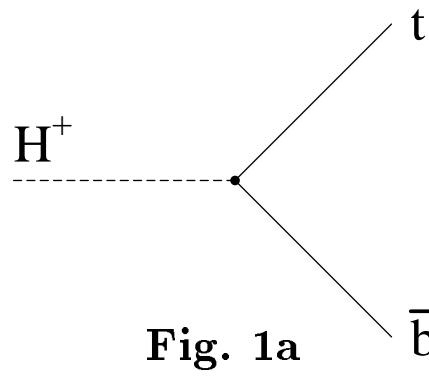


Fig. 1a

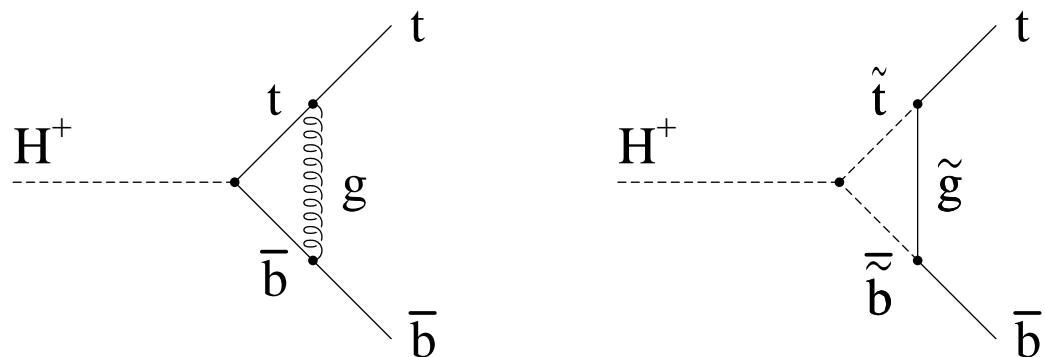


Fig. 1b



Fig. 1c

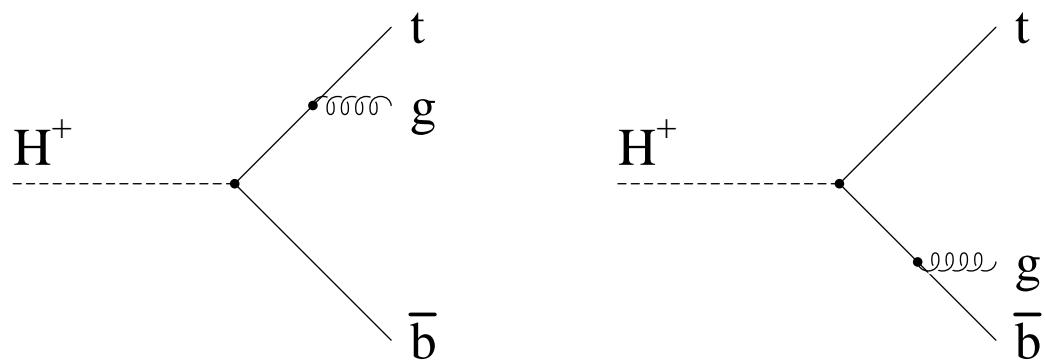
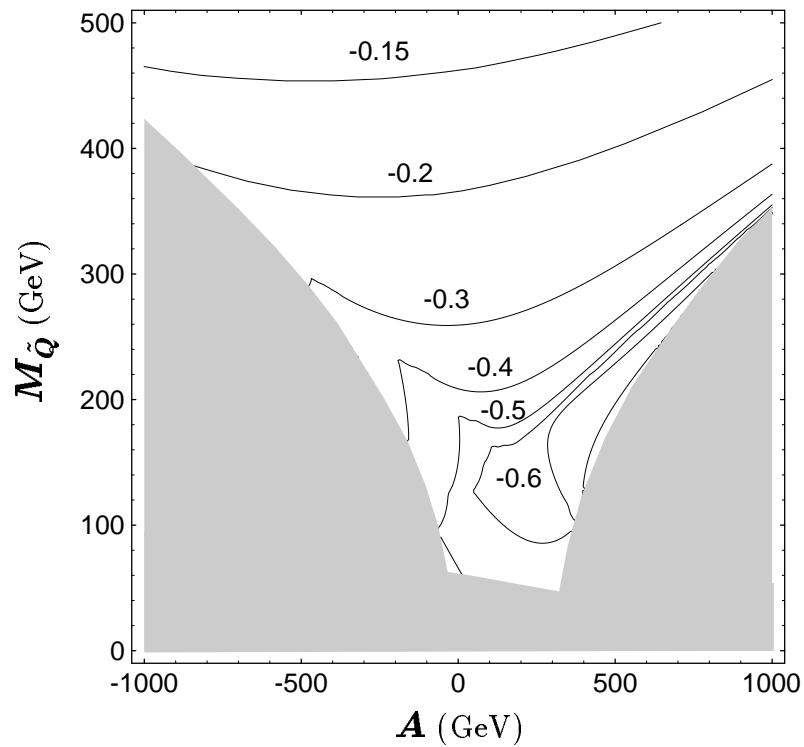
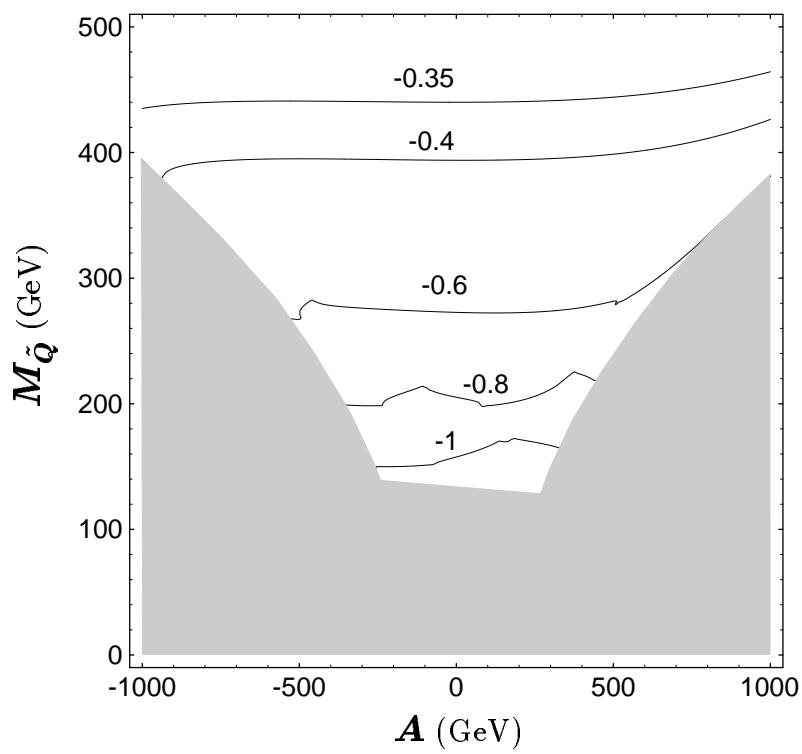


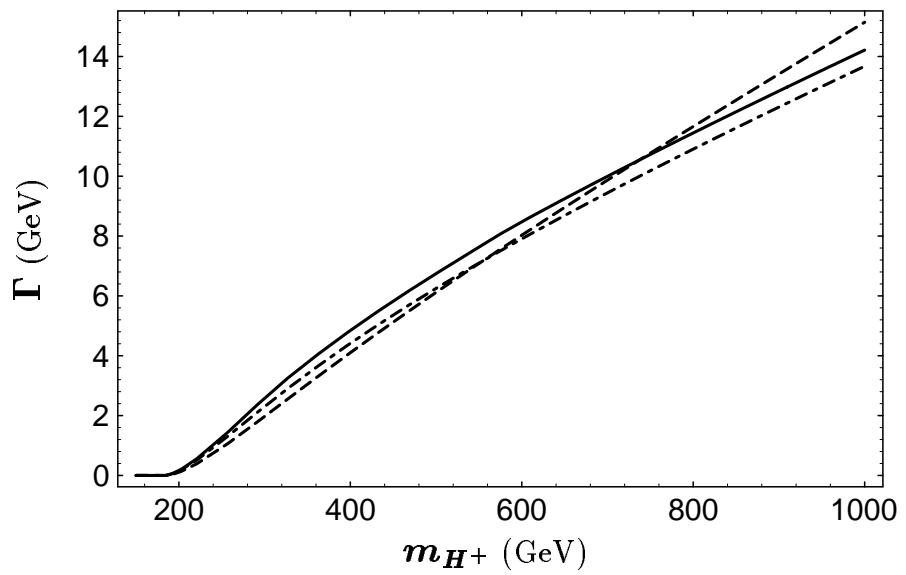
Fig. 1d



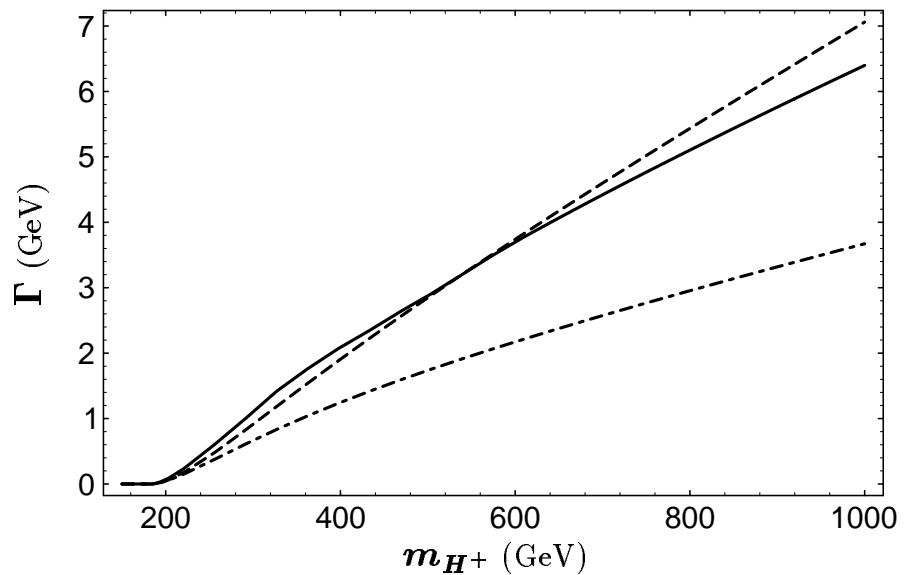
**Fig. 2a**



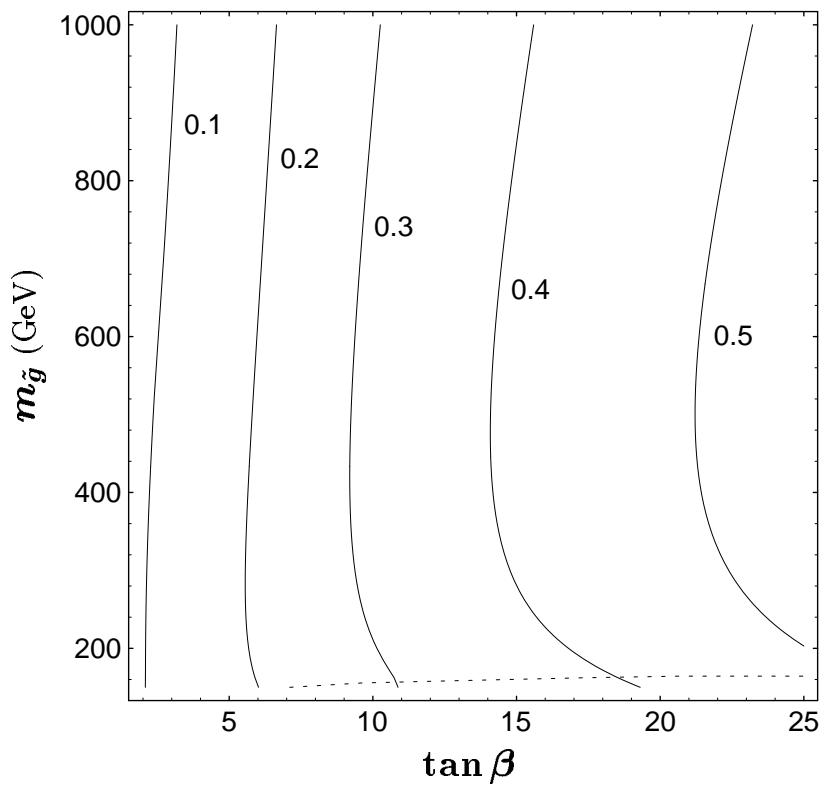
**Fig. 2b**



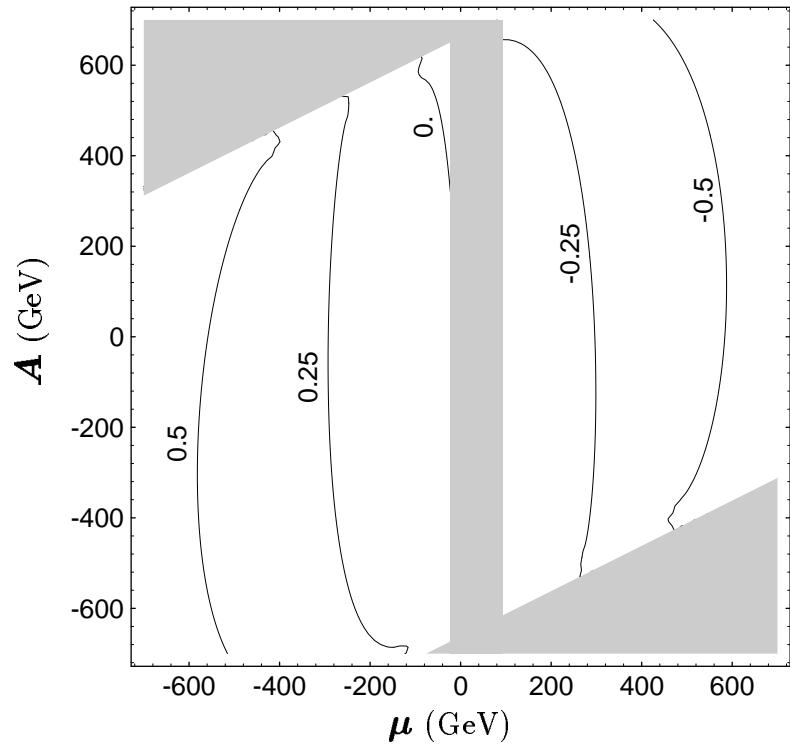
**Fig. 3a**



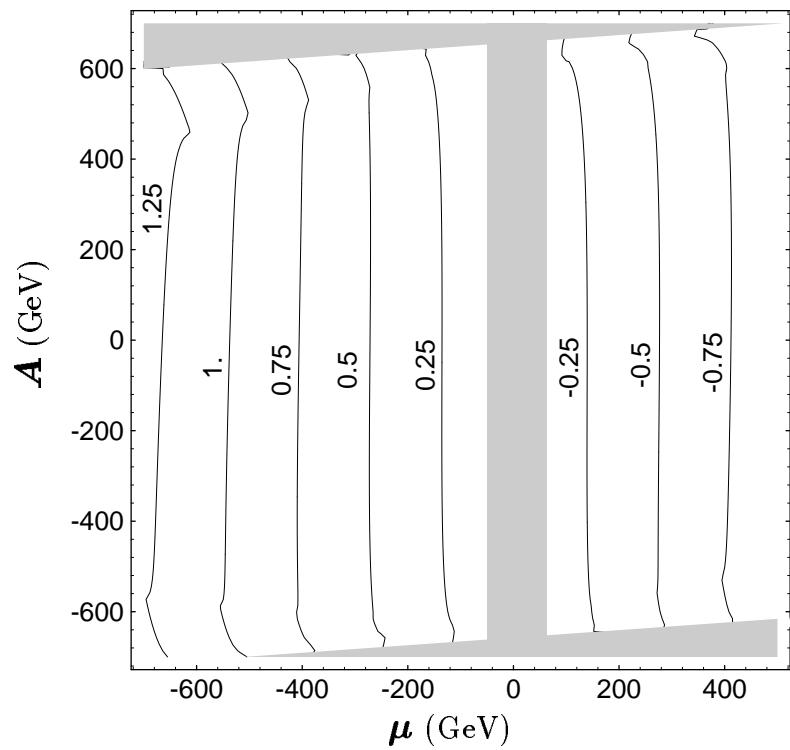
**Fig. 3b**



**Fig. 4**



**Fig. 5a**



**Fig. 5b**